

Bartók et le nombre d'or

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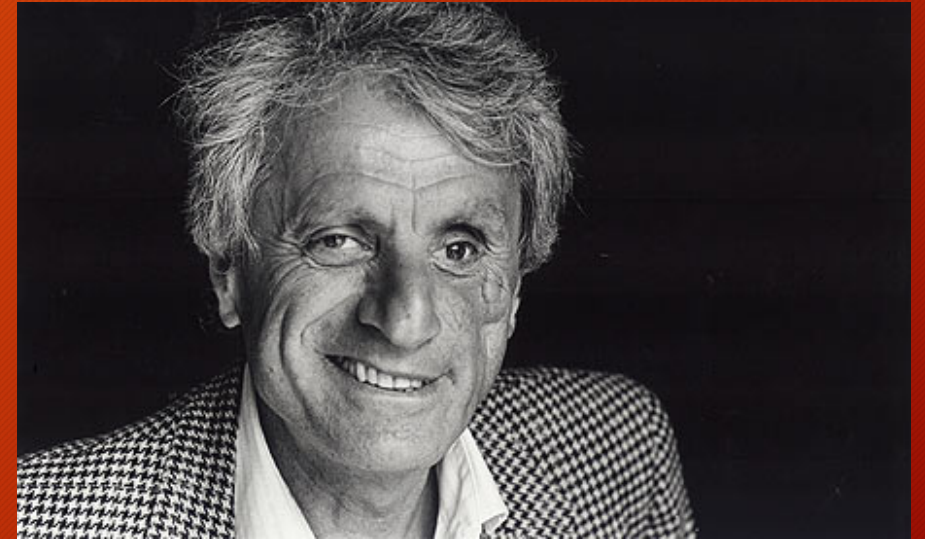
Club mathématique de l'Université de Montréal, 17 janvier 2018

« *La musique est un exercice
d'arithmétique secrète où l'esprit ne
réalise pas qu'il compte.* »

-Gottfried Wilhelm Leibniz (1646-1716)

Iannis Xénakis (1922-2001)

- Compositeur grec, qui a vécu en France
- Architecte auprès de Le Corbusier
- Souhaite formaliser la composition musicale: élabore une « axiomatique musicale »
- *Musiques formelles* (1963)
- Utilise des lois de probabilités pour créer de la musique « aléatoire »



[1] Iannis Xénakis, *Musiques formelles*, Éditions Richard-Masse, 1963

À paraître...

Iannis Xénakis: Musique aléatoire et mathématiques

Guillaume Laplante-Anfossi, Université de Montréal

Sous la supervision de

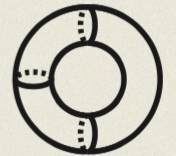
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Résumé

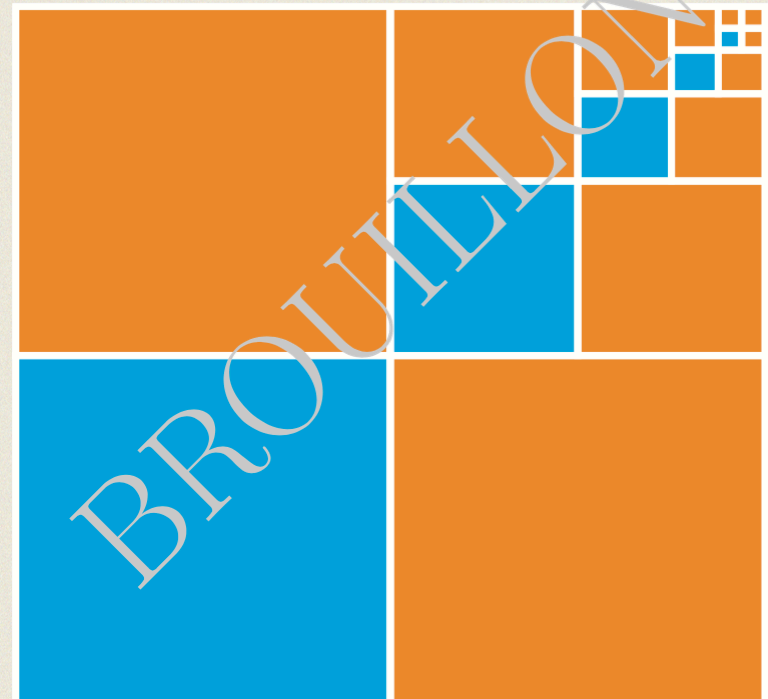
Iannis Xénakis (1922-2001) est sans doute l'un des compositeurs les plus importants du XXe siècle. Architecte et ingénieur de formation, il entrevoit à travers l'essor de la théorie des probabilités de grandes possibilités créatrices. Son projet est ambitieux : à la manière de Hilbert qui souhaitait au début du siècle axiomatiser la physique, Xénakis cherche à formaliser la composition musicale. Dans son livre *Musique formelles* [Xé63], il articule une démarche de création basée sur les mathématiques, manipulant des «grains de sons» à l'aide de lois de probabilités. Nous nous proposons ici d'introduire le personnage et sa démarche à travers la construction de l'une de ses premières oeuvres de «musique stochastique», *Achorripsis* [Xé58][Tam09]. Notre présentation sera axée sur le processus concret de composition, sur le *comment* des mathématiques en musique, dans le but de donner une nouvelle dimension à l'écoute des oeuvres du compositeur.

Nous débutons par un bref portrait biographique du compositeur. Après un rappel de quelques notions d'acoustique musicale et de probabilités, nous définissons sa démarche par

J É M U M



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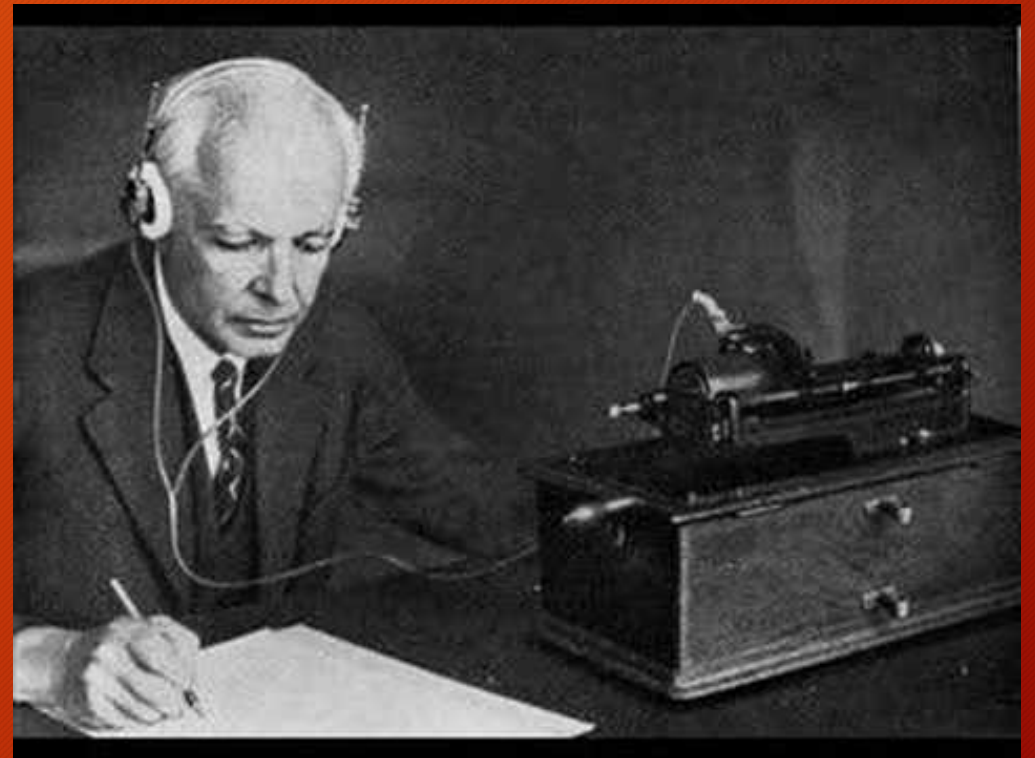
David Hilbert (1862-1943)



- On ne peut pas seulement se fier à l'intuition, il faut fonder les mathématiques sur des bases rigoureuses -*axiomatiques*
- *Grundlagen der Geometrie* (1899)
- 23 problèmes (1900)
- Le programme de Hilbert: « Nous devons savoir, nous saurons » (1930)

Béla Bartók (1881-1945)

- Pianiste et compositeur hongrois
- Étude extensive du folklore de son pays (ethnomusicologie)
- Plus d'une centaine d'œuvres (piano, musique de chambre, orchestre)
- Musique « organique » et « polymodale »
- Trouve son inspiration dans la Nature



Conférences de Harvard (1943)

« Du reste, l'élaboration [...] est survenue de manière subconsciente et instinctive elle aussi. Je n'ai jamais créé de théories à l'avance. Je détestais les idées de la sorte.

[...] Cette attitude ne signifie pas que je composais... sans plans définis et sans contrôle suffisant. [...]

Lorsque je m'apprêtais à écrire une pièce, je ne me suis jamais préoccupé de théories générales que j'aurais dû lui appliquer. »

-

[2] Béla Bartók, *Écrits*, Éditions Contrechamps, 2006

L. E. J. Brouwer (1881-1966)



- Élabore une philosophie des mathématique fondée sur l'intuition
- Refuse le raisonnement par l'absurde et le principe du tiers exclu (on parlera de *logique intuitionniste*)
- Henri Poincaré (1854-1912) défendra également une forme d'intuitionnisme

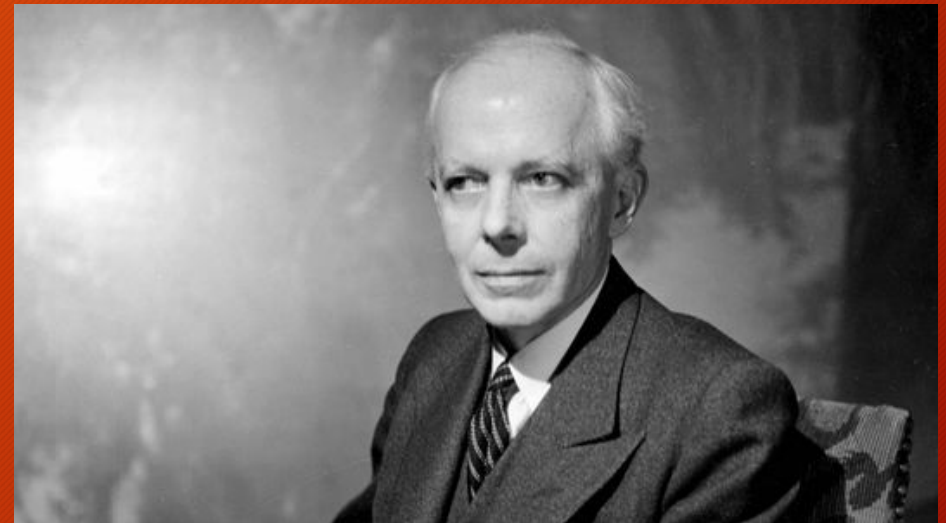
Une première écoute...

Musique pour cordes, percussions et célesta (1936)

II. Allegro



III. Adagio



Une première écoute... (suite)

III. Adagio



66

III.

Adagio, ♩ ca 66

allarg. - -

Timpani

Xylophon

mf

rubato

mf

p

5 - - al - Adagio molto, ♩ ca 40

Timp.

Xyl.

dim.

pp



Le nombre d'or

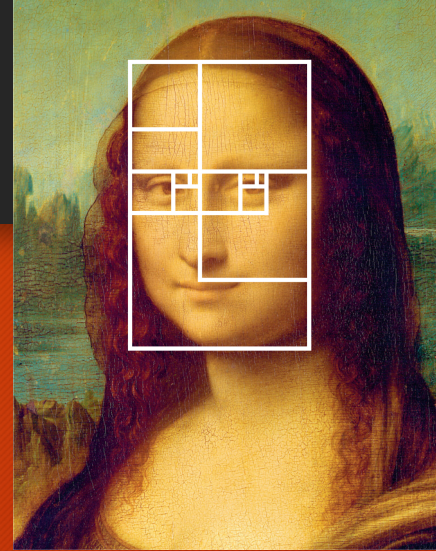
- Nombre réel positif (irrationnel)

$$\phi = \frac{1 + \sqrt{5}}{2}$$

$$a, b \in \mathbb{R}_{>0} \quad \frac{a}{b} = \frac{a+b}{a} := \phi$$

- Définition algébrique: unique solution positive de l'équation quadratique

$$x^2 - x - 1 = 0$$



- Définition géométrique: rapport particulier entre deux longueurs

Le nombre d'or

- Définition via la suite de Fibonacci

$$\{F_n\}_{n \in \mathbb{N}}$$

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2}$$

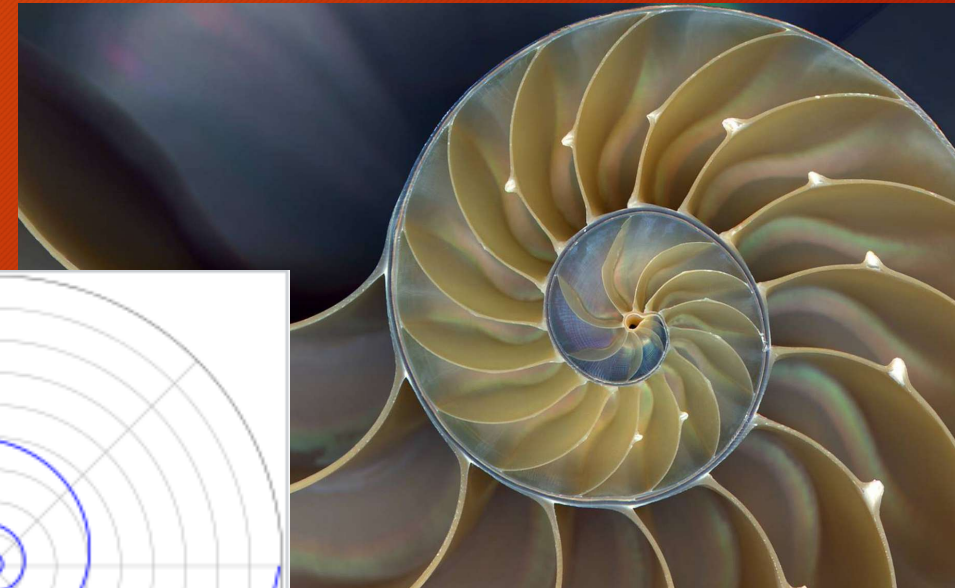
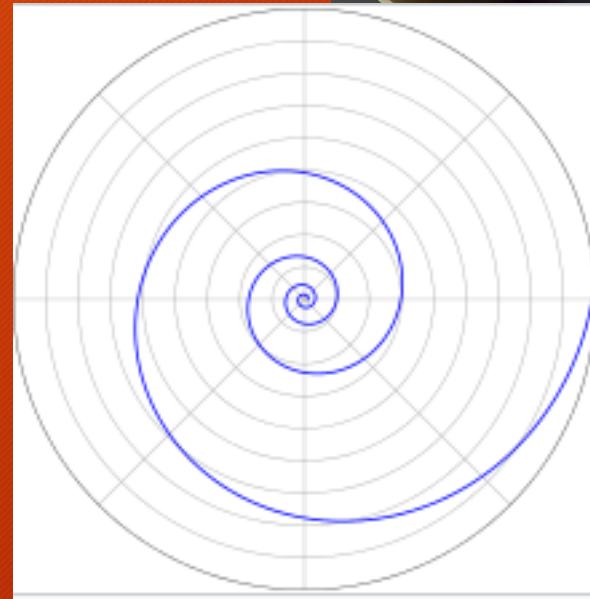
$$\phi = \lim_{n \rightarrow \infty} \frac{F_{n+1}}{F_n}$$



Spirale logarithmique

- Construite à partir du nombre d'or
- Étudiée par Jacques Bernoulli (1654-1705) pour ses propriétés d'invariance
- On la retrouve à quelques endroits dans la Nature...

$$r = ab^{\theta} \quad a, b \in \mathbb{R}, b \neq 1$$

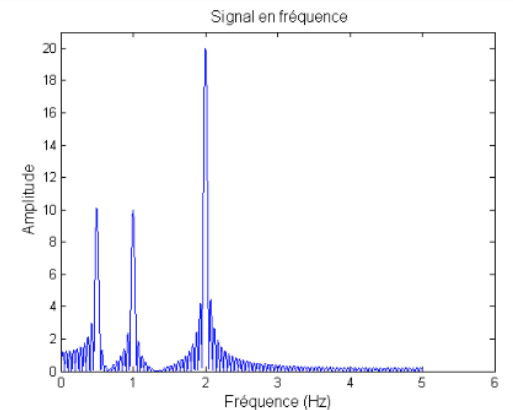
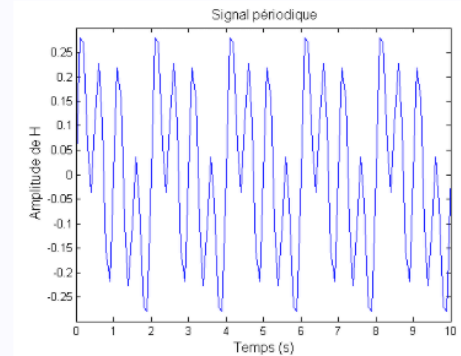


Quelques notions d'acoustique

Un son musical est une onde mécanique qui se propage dans l'air.

- Hauteur = Fréquence (Hz)
- Intensité = Amplitude (Décibels)
- Durée = Temps d'émission (Secondes)
- Timbre = Harmoniques et formants (Distingue les instruments entre eux)

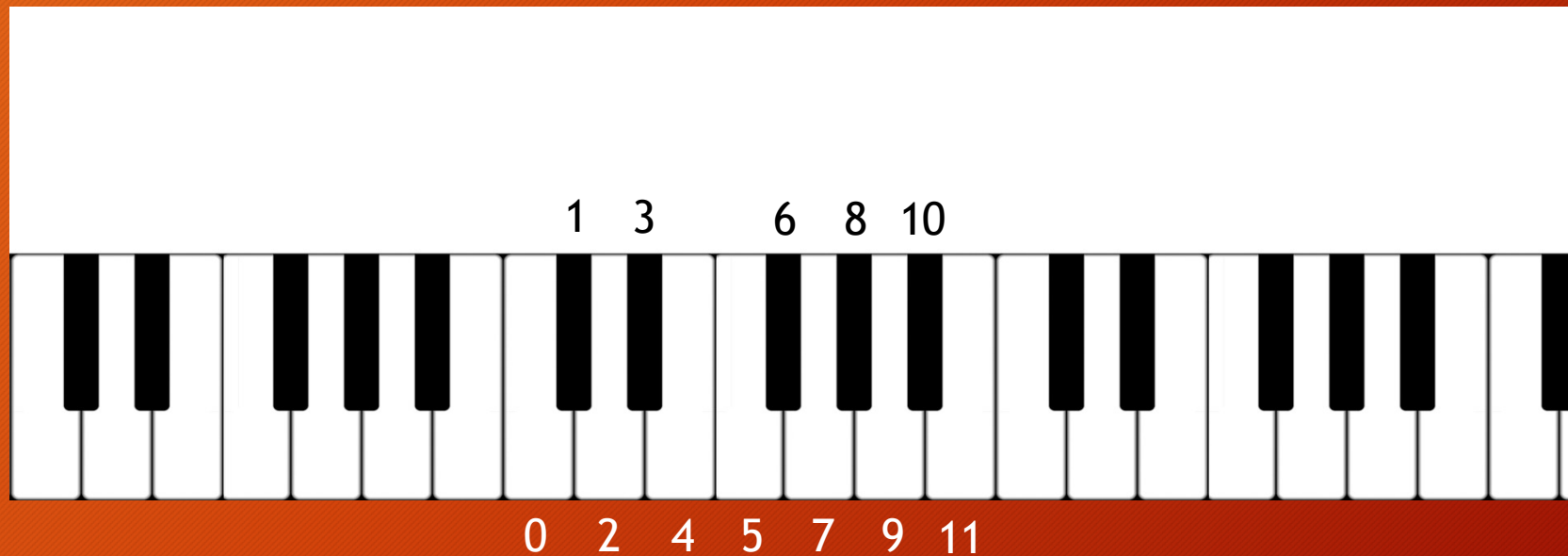
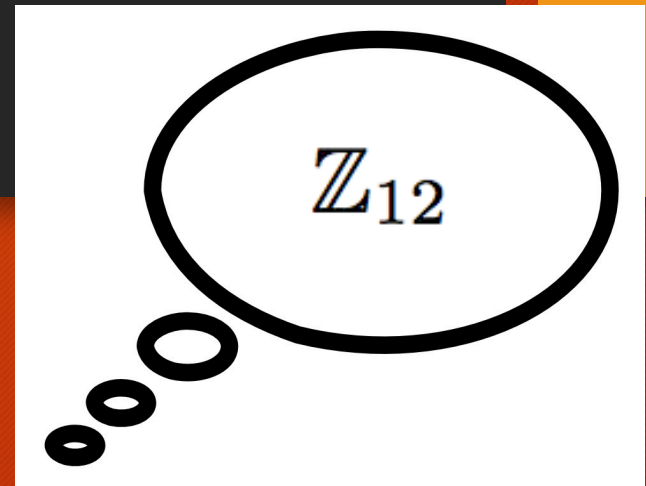
[3] Ulrich Michels, *Guide illustré de la musique*, Fayard, 1988



Un peu de théorie musicale

Qu'est-ce que la gamme tempérée?

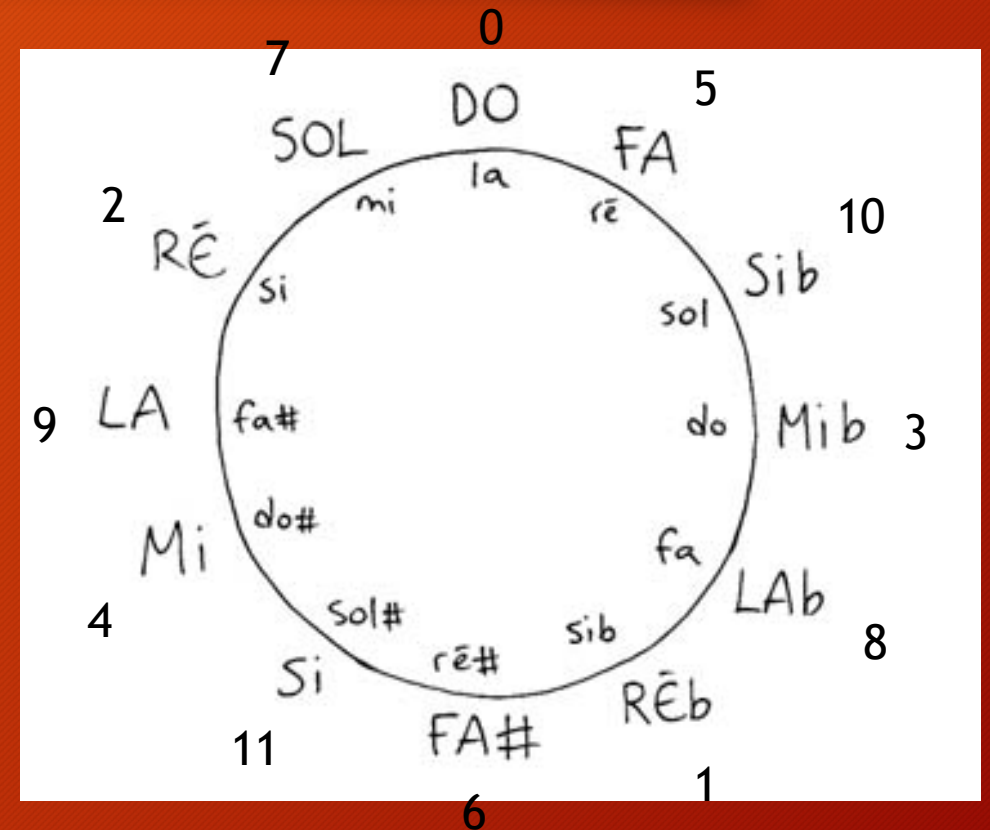
- Division de l'octave en 12 demi-tons égaux (racine 12^e de 2)



Le cycle des quintes

$$(7, 12) = 1 \implies O(\bar{7}) = 12$$
$$\implies \langle \bar{7} \rangle = \{\bar{7}, \bar{14} = \bar{2}, \bar{9}, \dots\} = \mathbb{Z}_{12}$$

- À la base que ce que l'on nomme la musique tonale
- «De Bach à Wagner»
- Et aussi toute la musique populaire d'aujourd'hui...



Le cycle des quintes

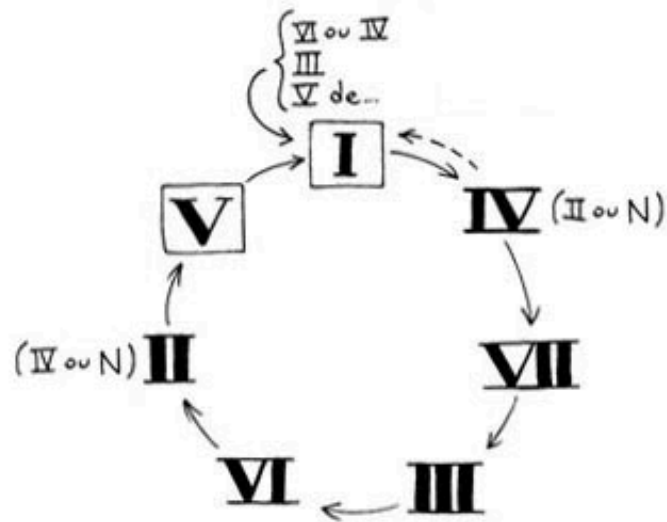
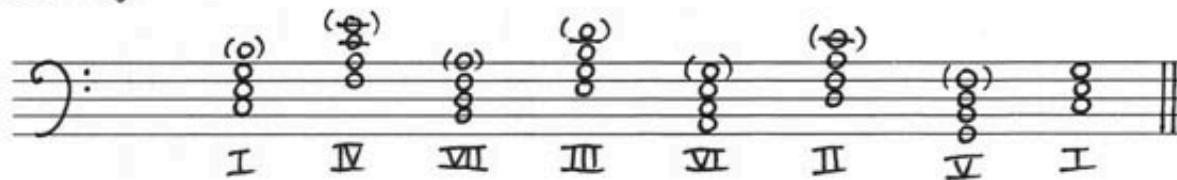


Figure 3

en majeur



- Les accords sur les 7 degrés de la gamme sont à la base de l'harmonie
- Chaque note entretient une relation *fonctionnelle* avec l'harmonie qui la sous-tend
- Jean Dieudonné (1906-1992): Les mathématiques ne s'intéressent pas à la *nature* des objets mais aux *relations* qu'ils entretiennent entre eux.

Les analyses de Ernő Lendvai (1925-1993)

Le système d'axes

- Dans la musique tonale, il y a une hiérarchie entre les degrés. Le plus fort est le 1, suivi de 5 et 4.
- Lendvai introduit une *relation d'équivalence* (fonctionnelle) entre les notes constituant un même axe
- Importance du triton («tonique» => «pôle»)
- Il s'agit pour lui d'une extension de la tonalité.

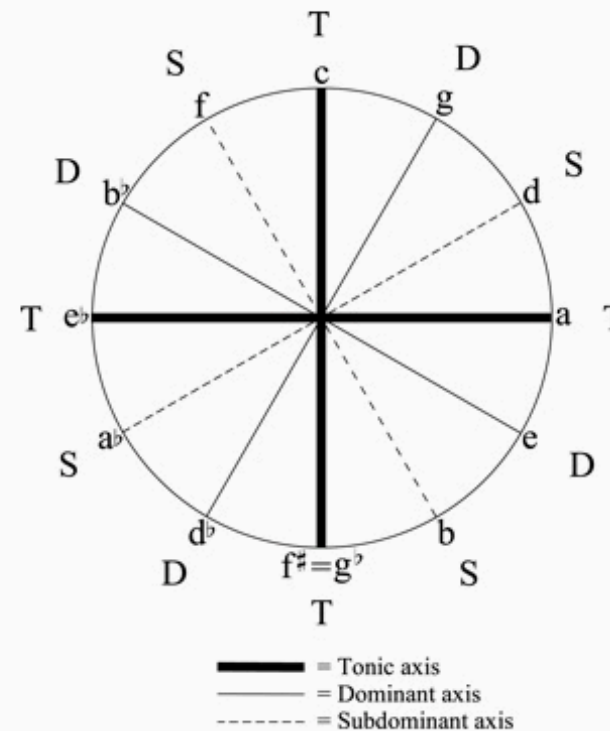
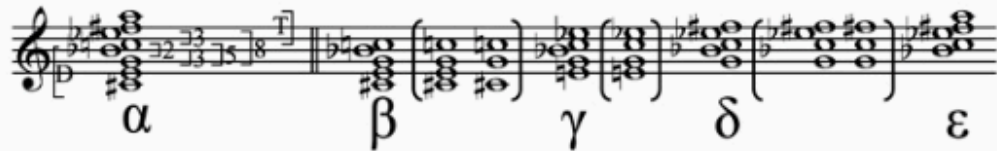


Figure 14.1 The circle of fifths with Lendvai's three axes.

Les analyses de Ernő Lendvai (1925-1993)

Example 14.2

(a) Lendvai's chord and its segments as explained in Lendvai, Béla Bartók: An Analysis of his Music, pp. 42–66, and in figure 37, p. 44



(b) Lendvai's 1:2, 1:3, and 1:5 models



La mélodie

- Les modes utilisés par Bartók sont régis par les premiers nombres de la suite de Fibonacci

Example 14.1 Pe



- Certains accords récurrents, symétriques, s'expliquent aussi de cette manière

Les analyses de Ernő Lendvai (1925-1993)

Les proportions

- Les événements musicaux significatifs sont séparés par un nombre de mesures ou de temps correspondant à Fibonacci
- Le moment de plus grande intensité est au germe d'une spirale logarithmique
- Disposition symétrique des plus grandes sections de part et d'autre du *fff*

[5] Ernő Lendvai, *Béla Bartók, An Analysis of his Music*, Kahn & Averill, 1971.

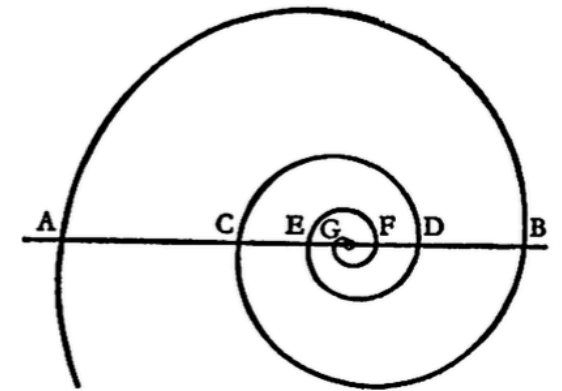


FIG. 25

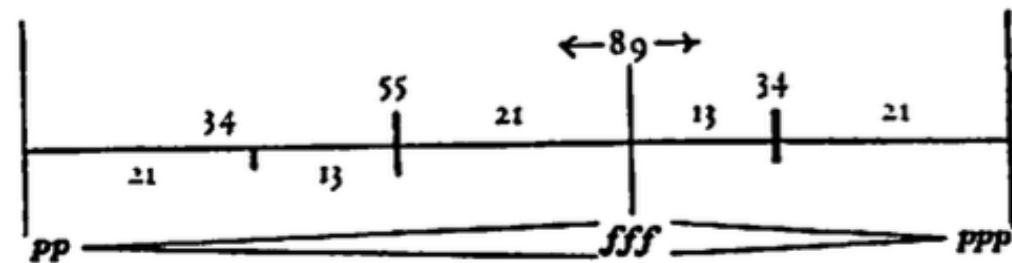
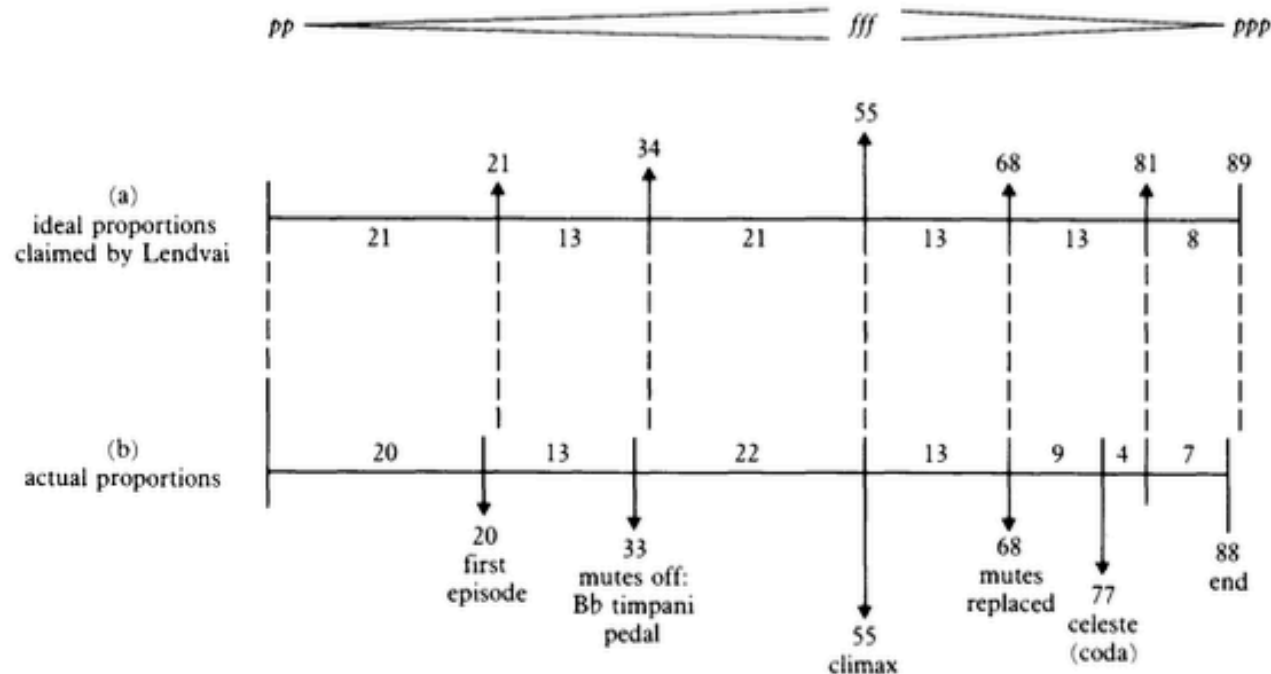


FIG. 22

Les analyses de Lendvai (suite)

Fig. 4: Fugue from Music for Strings, Percussion and Celeste



- Un examen plus attentif nous montre quelques approximations...
- Notre ami tourne quelques coins ronds!

[6] Roy Howat, *Bartók, Lendvai and the Principles of Proportional Analysis*, *Music Analysis* 2 (1), 1983, pp.69-95

Les analyses de Lendvai (suite)

- « Unfortunately, a good deal of doubtful arithmetic manipulation, practised above all by Lendvai, deflected attention from the essential [...] »

[7] Malcolm Gillies (ed.), *The Bartók Companion*, Amadeus Press, 1994.

- « Chacune des analyses de Lendvai peut ainsi être relativisée; il lui faut, en de nombreux endroits, tordre légèrement les chiffres pour parfaire sa démonstration. »

[8] Claire Delamarche, *Béla Bartók*, Fayard, 2011.

Certaines correspondances probantes

- Il fallait s'y attendre! Bartók ne compose pas avec un système de nombres; c'est une analyse *a posteriori*...
- Néanmoins, son sens des proportions est frappant, dans au moins quatre oeuvres de maturité

Musique pour cordes, percussions et célesta (1936), 1^{er} mouvement: climax à la mesure 55 sur un total de 88.

Quatuor à cordes #5 (1939), 2^e mouvement: point central de la forme en arche à la 138^e noire sur un total de 223.

the 1990s, the number of people in the United States who are 65 years of age or older has increased by 50 percent, and the number of people 75 years of age or older has increased by 75 percent. The number of people 85 years of age or older has increased by 150 percent. The number of people 95 years of age or older has increased by 300 percent. The number of people 100 years of age or older has increased by 500 percent. The number of people 105 years of age or older has increased by 1,000 percent. The number of people 110 years of age or older has increased by 2,000 percent. The number of people 115 years of age or older has increased by 4,000 percent. The number of people 120 years of age or older has increased by 8,000 percent. The number of people 125 years of age or older has increased by 16,000 percent. The number of people 130 years of age or older has increased by 32,000 percent. The number of people 135 years of age or older has increased by 64,000 percent. The number of people 140 years of age or older has increased by 128,000 percent. The number of people 145 years of age or older has increased by 256,000 percent. The number of people 150 years of age or older has increased by 512,000 percent. The number of people 155 years of age or older has increased by 1,024,000 percent. The number of people 160 years of age or older has increased by 2,048,000 percent. The number of people 165 years of age or older has increased by 4,096,000 percent. The number of people 170 years of age or older has increased by 8,192,000 percent. The number of people 175 years of age or older has increased by 16,384,000 percent. The number of people 180 years of age or older has increased by 32,768,000 percent. The number of people 185 years of age or older has increased by 65,536,000 percent. The number of people 190 years of age or older has increased by 131,072,000 percent. The number of people 195 years of age or older has increased by 262,144,000 percent. The number of people 200 years of age or older has increased by 524,288,000 percent. The number of people 205 years of age or older has increased by 1,048,576,000 percent. The number of people 210 years of age or older has increased by 2,097,152,000 percent. The number of people 215 years of age or older has increased by 4,194,304,000 percent. The number of people 220 years of age or older has increased by 8,388,608,000 percent. The number of people 225 years of age or older has increased by 16,777,216,000 percent. The number of people 230 years of age or older has increased by 33,554,432,000 percent. The number of people 235 years of age or older has increased by 67,108,864,000 percent. The number of people 240 years of age or older has increased by 134,217,728,000 percent. The number of people 245 years of age or older has increased by 268,435,456,000 percent. The number of people 250 years of age or older has increased by 536,870,912,000 percent. The number of people 255 years of age or older has increased by 1,073,741,824,000 percent. The number of people 260 years of age or older has increased by 2,147,483,648,000 percent. The number of people 265 years of age or older has increased by 4,294,967,296,000 percent. The number of people 270 years of age or older has increased by 8,589,934,592,000 percent. The number of people 275 years of age or older has increased by 17,179,869,184,000 percent. The number of people 280 years of age or older has increased by 34,359,738,368,000 percent. The number of people 285 years of age or older has increased by 68,719,476,736,000 percent. The number of people 290 years of age or older has increased by 137,438,953,472,000 percent. The number of people 295 years of age or older has increased by 274,877,906,944,000 percent. The number of people 300 years of age or older has increased by 549,755,813,888,000 percent. The number of people 305 years of age or older has increased by 1,099,511,627,776,000 percent. The number of people 310 years of age or older has increased by 2,199,023,255,552,000 percent. The number of people 315 years of age or older has increased by 4,398,046,511,104,000 percent. The number of people 320 years of age or older has increased by 8,796,093,022,208,000 percent. The number of people 325 years of age or older has increased by 17,592,186,044,416,000 percent. The number of people 330 years of age or older has increased by 35,184,372,088,832,000 percent. The number of people 335 years of age or older has increased by 70,368,744,177,664,000 percent. The number of people 340 years of age or older has increased by 140,737,488,355,328,000 percent. The number of people 345 years of age or older has increased by 281,474,976,710,656,000 percent. The number of people 350 years of age or older has increased by 562,949,953,421,312,000 percent. The number of people 355 years of age or older has increased by 1,125,899,906,842,624,000 percent. The number of people 360 years of age or older has increased by 2,251,799,813,685,248,000 percent. The number of people 365 years of age or older has increased by 4,503,599,627,370,496,000 percent. The number of people 370 years of age or older has increased by 9,007,199,254,740,992,000 percent. The number of people 375 years of age or older has increased by 18,014,398,509,481,984,000 percent. The number of people 380 years of age or older has increased by 36,028,797,018,963,968,000 percent. The number of people 385 years of age or older has increased by 72,057,594,037,927,936,000 percent. The number of people 390 years of age or older has increased by 144,115,188,075,855,872,000 percent. The number of people 395 years of age or older has increased by 288,230,376,151,711,744,000 percent. The number of people 400 years of age or older has increased by 576,460,752,303,423,488,000 percent. The number of people 405 years of age or older has increased by 1,152,921,504,606,846,976,000 percent. The number of people 410 years of age or older has increased by 2,305,843,009,213,693,952,000 percent. The number of people 415 years of age or older has increased by 4,611,686,018,427,387,904,000 percent. The number of people 420 years of age or older has increased by 9,223,372,036,854,775,808,000 percent. The number of people 425 years of age or older has increased by 18,446,744,073,709,551,616,000 percent. The number of people 430 years of age or older has increased by 36,893,488,147,419,103,232,000 percent. The number of people 435 years of age or older has increased by 73,786,976,294,838,206,464,000 percent. The number of people 440 years of age or older has increased by 147,573,952,589,676,412,928,000 percent. The number of people 445 years of age or older has increased by 295,147,905,179,352,825,856,000 percent. The number of people 450 years of age or older has increased by 590,295,810,358,705,651,712,000 percent. The number of people 455 years of age or older has increased by 1,180,591,620,717,411,303,424,000 percent. The number of people 460 years of age or older has increased by 2,361,183,241,434,822,606,848,000 percent. The number of people 465 years of age or older has increased by 4,722,366,482,869,645,213,696,000 percent. The number of people 470 years of age or older has increased by 9,444,732,965,739,290,427,392,000 percent. The number of people 475 years of age or older has increased by 18,889,465,931,478,580,854,784,000 percent. The number of people 480 years of age or older has increased by 37,778,931,862,957,161,709,568,000 percent. The number of people 485 years of age or older has increased by 75,557,863,725,914,323,419,136,000 percent. The number of people 490 years of age or older has increased by 151,115,727,451,828,646,838,272,000 percent. The number of people 495 years of age or older has increased by 302,231,454,903,657,293,676,544,000 percent. The number of people 500 years of age or older has increased by 604,462,909,807,314,587,353,088,000 percent. The number of people 505 years of age or older has increased by 1,208,925,819,614,629,174,706,176,000 percent. The number of people 510 years of age or older has increased by 2,417,851,639,229,258,349,412,352,000 percent. The number of people 515 years of age or older has increased by 4,835,703,278,458,516,698,824,704,000 percent. The number of people 520 years of age or older has increased by 9,671,406,556,917,033,397,649,408,000 percent. The number of people 525 years of age or older has increased by 19,342,813,113,834,066,795,298,816,000 percent. The number of people 530 years of age or older has increased by 38,685,626,227,668,133,590,597,632,000 percent. The number of people 535 years of age or older has increased by 77,371,252,455,336,267,181,195,264,000 percent. The number of people 540 years of age or older has increased by 154,742,504,910,672,534,362,390,528,000 percent. The number of people 545 years of age or older has increased by 309,485,009,821,345,068,724,781,056,000 percent. The number of people 550 years of age or older has increased by 618,970,019,642,690,137,449,562,112,000 percent. The number of people 555 years of age or older has increased by 1,237,940,039,285,380,274,899,124,224,000 percent. The number of people 560 years of age or older has increased by 2,475,880,078,570,760,549,798,248,448,000 percent. The number of people 565 years of age or older has increased by 4,951,760,157,141,521,099,596,496,896,000 percent. The number of people 570 years of age or older has increased by 9,903,520,314,283,042,199,193,993,792,000 percent. The number of people 575 years of age or older has increased by 19,807,040

Concerto pour piano #3 (1945), 1^{er} mouvement:
réexposition débute exactement à la 342^e noire
sur un total de 554.

Proportions du 1° Mvt : Sections positives : AB AØ1 AØ2

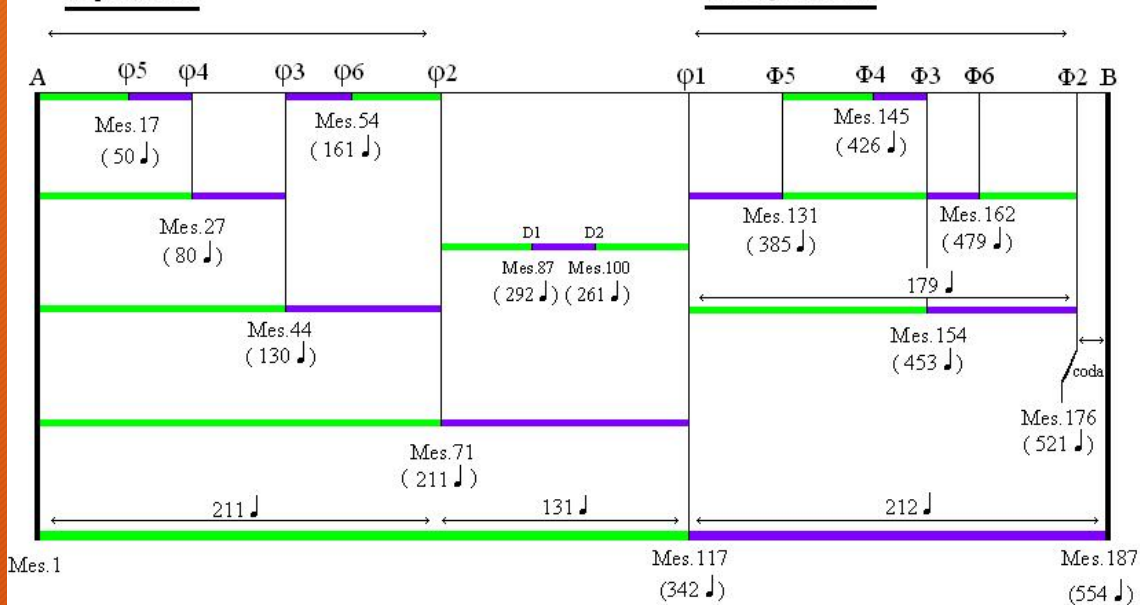
L'unité est la Sections négatives : $\frac{AB}{A\phi_1} = \frac{A\phi_2}{A\phi_3} = \frac{A\phi_4}{A\phi_5}$ etc ..

Sections négatives :

$$\frac{AB}{A\phi 1} = \frac{A\phi 1}{A\phi 2} = \frac{A\phi 2}{A\phi 3} \text{ etc.}$$

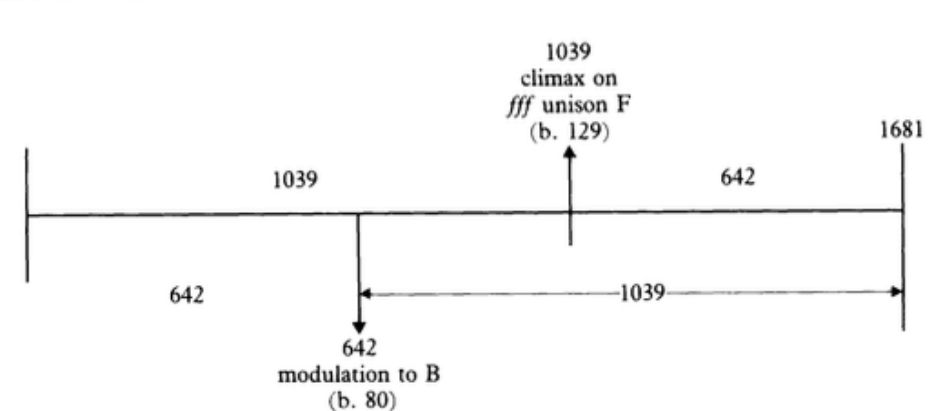
Exposition : Réexposition :

Réexposition :



Divertimento pour orchestre à cordes (1939), 1^{er} mouvement: climax à la 1039^e croche sur 1681; modulation importante à la 642^e=(1681-1039)^e croche!

Fig. 2: first movement of *Divertimento*



[6] Roy Howat, *Bartók, Lendvai and the Principles of Proportional Analysis*, Music Analysis 2 (1), 1983, pp.69-95

Certaines correspondances probantes

- « [...] rien ne laisse supposer qu'il se passionnait pour le nombre d'or, ni même en connaissant l'existence. Le sens des proportions de Bartók n'en est pas moins troublant. Mais, à l'évidence, il naît d'une pure intuition. »

[8] Claire Delamarche, *Béla Bartók*, Fayard, 2011.

- «this fugue [...] is an excellent example of natural growth principles in action but that there is no evidence that Bartók was aware of these principles or designed any of his works according to them. The sketches to his later works make it clear that the music just grew this way in his hand. »

[7] Malcolm Gillies (ed.), *The Bartók Companion*, Amadeus Press, 1994.

Dans l'atelier du compositeur



- De manière générale, le compositeur travaille avec un matériau de base, sur lequel il *opère certaines transformations*
- Elles peuvent concerner chacun des 4 paramètres ou plusieurs à la fois: les hauteurs, les intensités, les durées, le timbre.

Dans l'atelier du compositeur

Les hauteurs

- Thème
- Inversion
- Rétrogradation
- Inverse-rétrograde



The image displays four musical staves, each representing a different transformation of a theme. Each staff begins with a treble clef, a key signature of one sharp (F#), and a time signature of 8/8. The tempo is marked as 114. The first staff shows the original theme, starting with a whole note G4, followed by quarter notes A4, B4, C5, and a half note D5. The second staff shows the inversion, where the intervals are mirrored around a central axis. The third staff shows the retrograde, where the theme is played backwards. The fourth staff shows the inverse-retrograde, which is the retrograde of the inverted theme. Each staff is divided into two measures by a double bar line, with a 12/8 time signature change indicated at the beginning of the second measure.



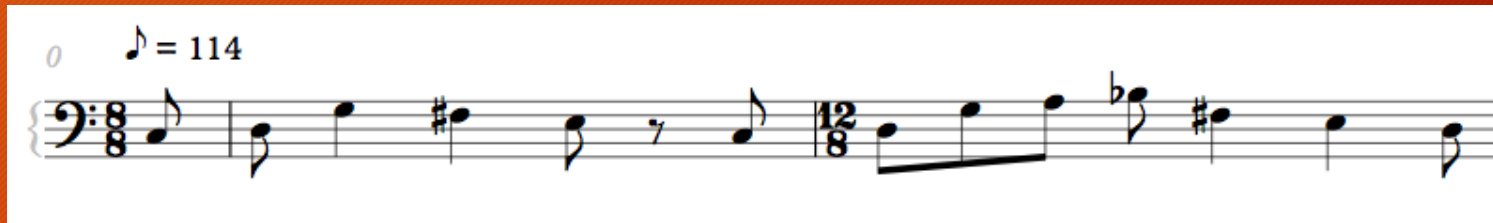
Dans l'atelier du compositeur

Les hauteurs

- Transposition



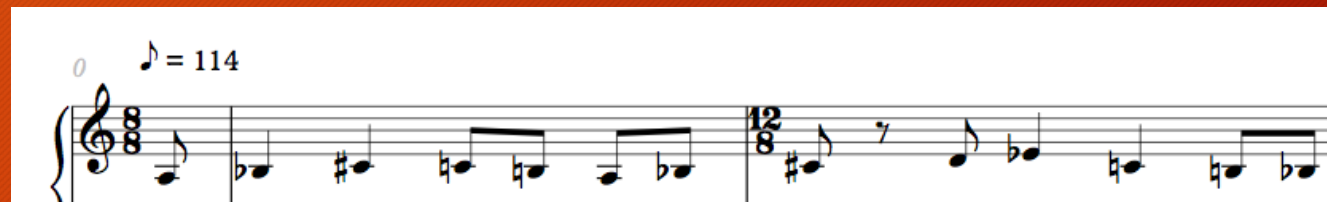
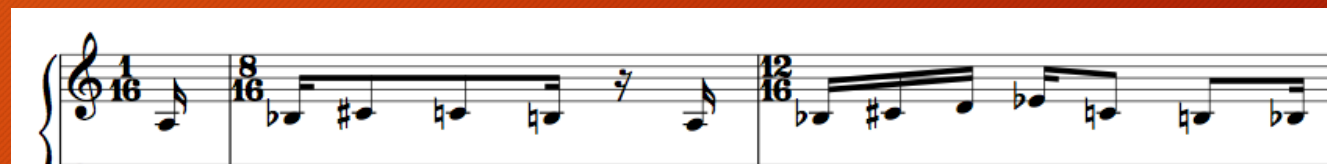
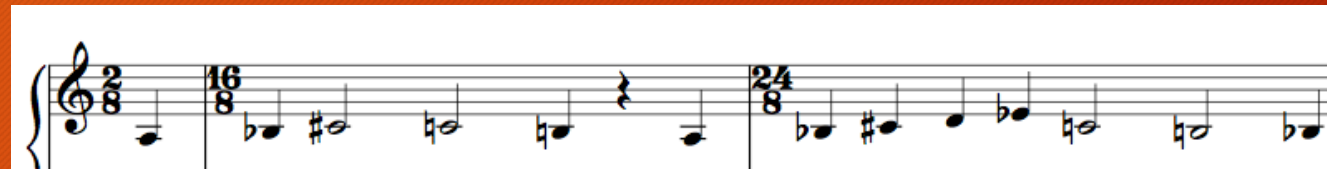
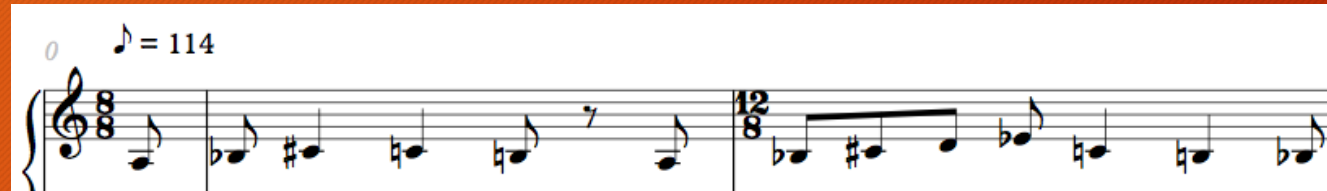
- Extension «à la Bartók»



Dans l'atelier du compositeur

Les durées

- Thème
- Augmentation
- Diminution
- Rétrogradation



Les symétries

- C'était clairement une tendance chez Bartók...
- Forme en arche
- Transformation des motifs, variation perpétuelle
- Écoutons le premier mouvement de *la Musique pour cordes, percussion et célesta* (1936)
- A été utilisé pour la bande sonore du film *The Shining* (Stanley Kubrick, 1980)
- Schéma au tableau: triton chromatique thème, fugue en quintes successives, climax sur l'axe, thème en inversion, forme en arche, nombre d'or.

<https://www.youtube.com/watch?v=m129k5YcQnU>

[7] Malcolm Gillies (ed.), *The Bartók Companion*, Amadeus Press, 1994.

Conclusion

*« La musique est un exercice
d'arithmétique secrète où l'esprit ne
réalise pas qu'il compte. »*

-Gottfried Wilhelm Leibniz (1646-1716)

Merci de votre écoute!

Des questions ?! ...

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- [8] Claire Delamarche, *Béla Bartók*, Fayard, 2011.
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Quelques suggestions d'écoutes!

- Contrastes
- 5^e quatuor à cordes
- 3^e Concerto pour piano
- Concerto pour orchestre
- Musique pour cordes, percussion et célesta
- Divertimento
- Sonate pour deux pianos et percussions